Exploring the effects of perception errors and anticipation strategies on traffic accidents - a simulation study.

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Abstract. It is remarkable that drivers (on average) can safely navigate through dense traffic at high speeds—conditions in which the time headways between vehicles are in the same order of magnitude as human reaction times. One explanation for this is the ability of drivers to anticipate on the traffic conditions in their surroundings. In this paper, we study, through simulation, the effects of reaction times, errors in perception and anticipation on the probability of accidents on freeways. To this end we extend an existing model for car following and lane changing with a perception and anticipation model inspired by Ensley's three levels of situational awareness (perception, understanding and projection). By systematically varying driving behavior with different reaction times over a range of perception errors, and anticipation strategies, we compute efficiency effects (capacity and total time spent) and safety effects (the probability density of accidents happening as a function of these different contributing factors and errors). The results provide some evidence that safe driving is robust with respect to perception errors under simple anticipation strategies and small reaction times. When reaction times grow larger, more advanced anticipation strategies are needed to guarantee safe driving.

Keywords: Driving behavior · Awareness · Perception errors · Anticipation strategies · Traffic safety

1 Introduction

On a yearly basis 1.5 Million people get killed and several tens of millions get severely injured in traffic incidents worldwide [1]. Even in countries with the highest road design and regulation standards, such as The Netherlands (570 deaths and 19,000 injured in 2015 [2]), the total economic loss due to traffic unsafety is estimated at € 12.5 billion, or 2.2% of the Gross Domestic Product (GDP), which is more than four times the total delay costs of congestion [3].

In this paper, we focus on the effects of perception errors and anticipation strategies on traffic unsafety. With human reaction times in the same order of magnitude as time headways at capacity flows (between 0.5 and 2 seconds), *it is remarkable that* *drivers can safely maintain high speeds through dense traffic at all*. Even more so when considering the many perception errors humans typically make, e.g. in assessing relative distances and velocities of vehicles close by. One explanation for this phenomenon is the ability of drivers to anticipate on the traffic conditions further downstream and to predict the movement of vehicles in their direct surroundings in the short-term future. This mechanism is well known in control theory and engineering: prediction has a stabilizing effect on control systems with delayed input. However, the intricate balance between reaction time and anticipation is fragile as the sobering statics at the start of this paper confirm. Moreover, whereas perception can be enhanced through technology and perception errors can be automated largely "out of the equation", automation of anticipation and prediction is a much harder problem solve, because it requires the sort of reasoning and intuition that comes natural to humans but is very difficult to capture in mathematical models.

In this paper, we study, through simulation, the effects of reaction times, errors in perception and anticipation strategies on the probability of accidents on free-ways. To this end we extend an existing "collision-free" model for car following and lane changing (the LMRS model) with a perception and anticipation model inspired by Ensley's three levels of situational awareness (perception, understanding and projection). By systematically varying driving behavior with different reaction times over a range of perception errors, anticipation strategies and errors therein, we compute capacity and several safety indicators as a function of these different contributing factors and errors (other contributors such as risk-taking or failing technology beyond the scope of this paper).

2 Overall simulation logic

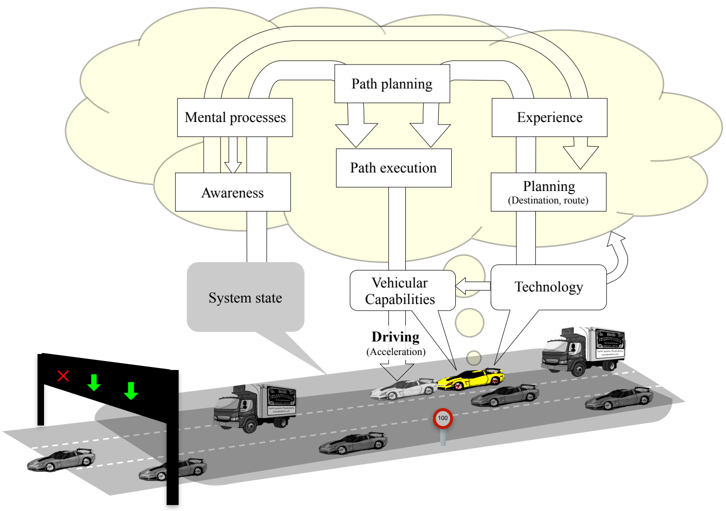

Fig. 1: Schematic delineation of the driving process as modeled in OpenTrafficSim.

Fig. 1 delineates the driving process as modeled in our open-source traffic simulator OpenTrafficSim (OTS) [4]. The overall simulation logic in OTS is that a driver—traveling with a certain strategical plan (a route) on a road stretch—subjectively perceives a portion of the surrounding traffic state. This perception process—potentially affected by the drivers' mental state, preferences, etc—results in a reconstruction of the relevant (instantaneous) state variables and an extrapolation of these into the near future. Using this (anticipated and possibly erroneous) personal state estimate the driver now plans a short-term path (i.e. a trajectory for the next 10-30 seconds) and executes it in terms of car following or overtaking. Note that OTS does not constrain or impose any specific mathematical model for any of these components.

We will not use the full conceptual OTS model in this paper. First, we will impose errors on perception without an explicit (dynamic) causal model that relates for example workload or other explanatory mental constructs to these errors. Furthermore, we do not consider strategic planning or learning; we do not consider path-planning and execution as separate processes nor do we use a separate vehicle model.

3 Mathematical model for driving behavior

Research into traffic flow theory dates back to the 1930s [5, 6] and matured in the 1950s [7, 8] when the first mathematical models for longitudinal driving were developed. Since then many schools of thought have emerged, each characterized by different behavioral assumptions and different ranges of descriptive and (partial) explanatory power for the resulting phenomena. For example, safe-distance models [9-11] assume that drivers maintain a large enough distance headway in case the leader brakes at maximum deceleration; optimal velocity models [12] assume that drivers accelerate to their optimal velocity as a function of the distance headway; whereas approaches in the more general group of stimulus-response models [13-15] make assumptions on how drivers adapt their speed on the basis of speed and headway and a range of additional factors. Incorporating perception and anticipation processes in traffic flow modelling is not new. Most models can straightforwardly be augmented to include reaction times; so-called psycho-spacing (or action point) models [16] incorporate drivers' inertia to observe and respond to small changes in stimuli; whereas multi-anticipatory models [17-19] include terms for anticipation of drivers to traffic conditions further downstream. A recent overview of models for longitudinal driving behavior can be found in [20]. A similar diversity of modelling approaches can be found for lateral driving behavior that governs when drivers change lanes, diverge, and merge [21-25]. What these models have in common is that they are—in principle—collision-free. This is no longer the case, however, if we allow for reaction times (i.e. delayed stimuli) and/or errors in these stimuli or both.

3.1 Lane change model

The model we employ is an Integrated Lane change Model with Relaxation and Synchronization (LMRS) [26]. This model offers a parsimonious and integrated approach to lane changing and reproduces several important freeway phenomena such as speed relaxation and synchronization, i.e. following vehicles in adjacent lanes. Although all these effects are captured, the lane change model has only 7 parameters. Below, we highlight the main rationale and refer to [26] for details. Most importantly, LMRS combines multiple lane change incentives into a lane change desire. The desire to change from lane i to lane j that arises from the different incentives is combined into a single desire d^{ij}, expressed as:

$$d^{ij} = d_r^{ij} + \theta_v^{ij}\left(d_s^{ij} + d_b^{ij}\right) \tag{1}$$

In Eq. (1), there is a desire to follow a route (d_r), to gain speed (d_s) and to keep right (d_b), where the subscript b stands for bias to a particular side (left or right). The latter two are included with θ_v which is the level at which voluntary (discretionary) incentives are included. Meaningful desires range between -1 and 1, where negative values indicate that a lane change is not desired (i.e. to stay or to change in the other direction). Values outside of the meaningful range may exist as incentives are added. The weight factor θ_v^{ij} is expressed as:

$$\theta_v^{ij} = \begin{cases} 0 \quad , & d_r^{ij} \cdot d_s^{ij} < 0 \text{ and } \left|d_r^{ij}\right| \geq d_{coop} \\ \frac{d_{coop} - \left|d_r^{ij}\right|}{d_{coop} - d_{sync}}, & d_r^{ij} \cdot d_s^{ij} < 0 \text{ and } d_{sync} \leq \left|d_r^{ij}\right| \leq d_{coop} d_r^{ij} \cdot d_s^{ij} \\ 1, & d_r^{ij} \cdot d_s^{ij} \geq 0 \text{ and } \left|d_r^{ij}\right| \leq d_{sync} \end{cases} \tag{2}$$

The weight factor implies that if both voluntary and mandatory lane-change desires are either negative or positive ($d_r \cdot dv \geq 0$), voluntary desire is fully included as it coincides with mandatory desire. However, if voluntary desire conflicts with mandatory desire ($d_r \cdot d_v < 0$), the voluntary desire is only partially included. The total desire determines the type of lane change behavior of drivers from three classes: Free Lane Changes (FLC), Synchronized Lane Changes (SLC) and Cooperative Lane Changes (CLC), identified by three thresholds of d_{free}, d_{sync}, d_{coop} as the model parameter:

$$0 < d_{free} < d_{sync} < d_{coop} < 1 \tag{3}$$

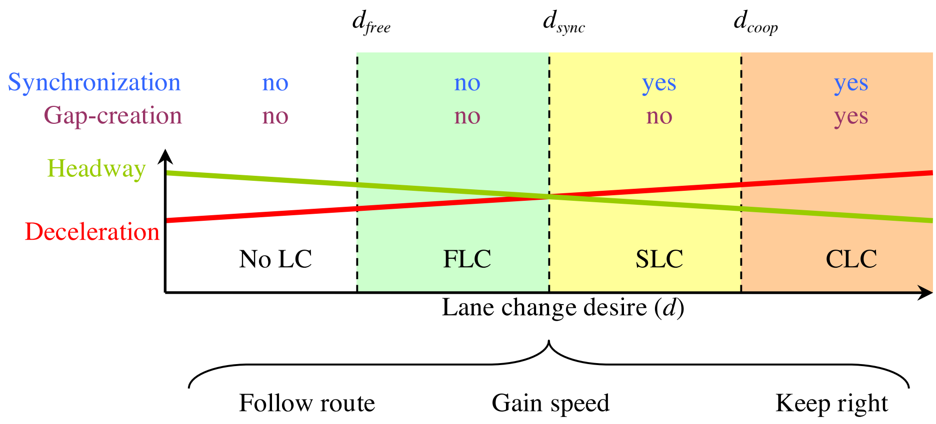

Fig. 2 Overview of LMRS. Lane change desire is based on three incentives. Lane change behavior, including the accepted headway and deceleration for a lane change, varies depending on the level of lane change desire [26].

Fig. 2 gives an overview of the variation of lane change behavior between processes. For little desire, no lane change will be performed. For a somewhat larger desire, FLC is performed requiring no preparation whatsoever. In SLC and CLC, a potential lane changing driver is willing to synchronize their speed with a vehicle on the target lane. This is achieved by following a vehicle in that lane. Concurrently, this will align the vehicle with a gap (if there is a gap); this is thus a simple gap-searching model. In CLC, the potential follower will additionally start to create a gap by following the potential lane changing vehicle.

3.2 Integration of lane change model and car following model

LMRS works with any car following model. Here we use a slightly adapted version of the Intelligent Driver Model (IDM) by Treiber et al. [14]. The acceleration is calculated with

$$\dot{v} = a \cdot \min\left(1 - \left(\frac{v}{v_{des}}\right)^4, 1 - \left(\frac{s^*}{s}\right)^2\right) \tag{4}$$

and

$$s^* = s_0 + v \cdot T + \frac{v \cdot \Delta v}{2\sqrt{a \cdot b}} \tag{5}$$

where s_0 is the stopping distance, Δv is the approaching rate to the leader, s is the gap (net distance headway) and s^* is the dynamic desired gap. The adapted model is referred to as IDM+ because it results in more realistic values for capacity. Integration of lane change and car following model takes place in the gap-acceptance process. In LMRS, a gap is accepted or rejected based on the resulting deceleration that follows from the car-following model. Gaps that result in deceleration that is too large, are rejected as they are unsafe, uncomfortable or impolite. The gap is accepted if both the lane changer (c) and the new follower (f) will have an acceleration that is larger than some safe deceleration threshold $-b^c$. Note that in Fig. 2, we hint that the acceptable headway changes as a function of the lane change desire. For larger lane change desires, larger decelerations and shorter headways are accepted. If the lane change is initiated, both vehicle c and f should update their desired time headway T. When vehicles accept smaller headways than their normal ones, they will gradually relax their headway towards the normal value exponentially with a relaxation time window τ.

When the lane change desire is above the synchronization threshold, drivers will start to synchronize their speed with the leader on the target lane by applying the car-following model. Drivers will apply a maximum deceleration of b, which is considered as a both comfortable and a safe deceleration rate. Finally, if an adjacent leader wishes to change lane with a desire above the cooperation threshold, a gap will be created. Gap creation is similar to synchronization and we again apply the car-following model with a limited deceleration b.

3.3 Modelling perception errors and reaction time

The original LMRS is a deterministic model without explicit time delay. It does not capture the errors and time delay in situation awareness.

Perception error formulation. Important variables (or stimuli) for driving decision-making are the (relative) positions, speeds and accelerations of surrounding vehicles in addition to the ego vehicle speed v. We distinguish the error formulation in these variables. We assume that the error in the ego vehicle speed v is negligeable due to direct feedback from the speedometer. For the errors of position x_s and speed v_s of surrounding vehicles, there is evidence that the error of x_s and v_s is related to distance [27], e.g. the further away the predecessor is, the more difficult to accurately estimate x_s and v_s, and thus the higher the error of s and Δv is. We model the error as a standard Wiener process $w(t)$. The errors in position s and speed v and acceleration $\dot{v}$ of a surrounding vehicle are formulated as:

$$\widehat{x_s} = x_s + \mathrm{w(t)r_s}x_s\alpha \tag{6}$$

$$\widehat{v_s} = v_s + \mathrm{w(t)r_v s}_s \tag{7}$$

$$\widehat{\dot{v}_s} = \dot{v}_s + \mathrm{w(t)r_{\dot{v}}}\dot{v}_s \tag{8}$$

where α = 1 for leaders, or α = -1 for followers. This creates persistence of over- or underestimation. The perceived speed is limited to be non-negative, i.e. $\widehat{v_s} \geq 0$. This Wiener process $w(t)$ has a probability distribution which is the same as the standard normal distribution $N(0, 1)$. However, there is auto-correlation over time. This auto-correlation is described by a value of τ = 20 s. The numerical update scheme is given in equation:

$$w(t) = \begin{cases} e^{-\frac{\Delta t}{\tau}} w(t - \Delta t) + \eta\sqrt{\frac{2\Delta t}{\tau}}, P = 1 \\ \eta,\ P = 0 \end{cases} \tag{9}$$

where P = 1 means the surrounding vehicle was perceived in the previous time step, and P = 0 otherwise. Random value η is drawn from the standard normal distribution. Note that, although the acceleration of surrounding vehicles is not directly as the input for car following models, it will be used as the anticipation strategies in the ensuring of this section.

Reaction time. Including time delay in the model is straightforward in this formulation. We again assume there is no time delay in perceiving the ego vehicle speed, but a fixed time delay t_r for perceiving relative positions, speeds and accelerations of surrounding vehicles:

$$\widehat{x_s}(t - t_r) = x_s(t - t_r) + \mathrm{w(t)r_s}x_s(t - t_r)\alpha \tag{10}$$

$$\widehat{v_s}(t - t_r) = v_s(t - t_r) + \mathrm{w(t)r_v s}_s(t - t_r) \tag{11}$$

$$\widehat{\dot{v}_s}(t - t_r) = \dot{v}_s(t - t_r) + \mathrm{w(t)r_{\dot{v}}}\dot{v}_s(t - t_r) \tag{12}$$

One of the powerful features of the OTS simulation environment is that neither reaction times nor scheduling frequency have to be expressed in multiples of the numerical time step with which the simulation is executed. Without going into detail, this results in reaction times that have a stochastic component.

3.4 Modeling anticipation strategies

We assume drivers can have the capability to compensate for reaction time t_r by means of one of three anticipation strategies.

- *None*: the simplest case where no anticipation is performed, behavioural stimuli are taken from $t - t_r$; Essentially this comes down to an occlusion of t_r seconds
- *Constant speed*: drivers assume other vehicles move at a constant speed, which is perceived at $t - t_r$, to estimate distance and relative speed at time t;
- *Constant acceleration*: drivers assume other vehicles move with a constant acceleration, which is perceived at $t - t_r$, to estimate distance and speed at time t.

4 Experimental setup

4.1 Road network and scenarios

The experiment is carried out on a motorway corridor with a two plus two lane merge with a lane-drop after the merge (see Fig. 3). This is a very common configuration, that furthermore demands both voluntary and mandatory lane-changes and extensive vehicle interaction, while maintaining a single flow. This means that there are two potential bottlenecks, one at the merge, which heavily depends on vehicle lane changing behaviour, and a second one at the lane-drop, which is obviously more severe. The corridor is 9 km in length in total with the other distances of the various sections indicated in Fig. 3.

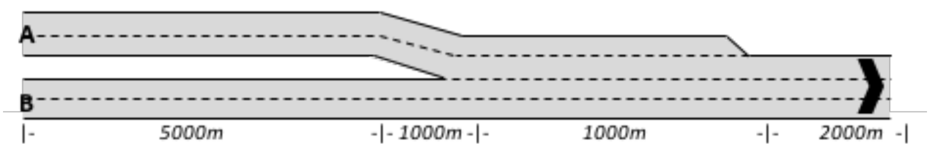

Fig. 3: Experimental network

A basic demand scenario is applied in the experiment that allows the influence of perception errors to be evaluated based on the three main traffic states: *free-flow, (near) capacity flow, and congested flow*. The demand distribution in time is shown in Fig. 4 and is given against the maximum flow on the two inflowing carriageways. The maximum flows on the two carriageways are 3500 veh/hr and 3200 veh/hr for carriageway A and B respectively. Furthermore, the distribution of generated vehicles for both carriageways over the two lanes is 55% for the left lane and 45% for the right lane, while 5% of all traffic are trucks, which are always generated on the right lane. The headways of inflowing traffic are exponentially distributed. There is a model warm-up time of 360 seconds with a demand set at 0.5 of the maximum flow.

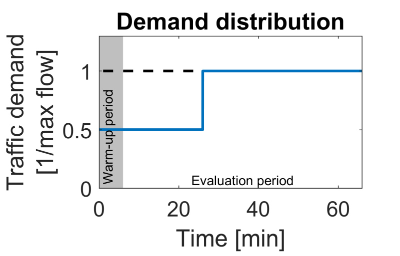

Fig. 4: Demand distributions for the basic demand scenario

For each anticipation strategy there are 36 scenarios by varying the error and reaction time. For the perception error we have r_s = {0.0, 0.02, 0.04, 0.06, 0.08, 0.1}. For the speed error we use $r_{\Delta v} = r_s$ / 5 = {0.0, 0.004, 0.008, 0.012, 0.016, 0.02}. Finally, for the acceleration error we use $r_{\dot{v}} = r_s \cdot 2 = \{0.0, 0.04, 0.08, 0.12, 0.16, 0.2\}$. While varying the extent of perception errors in our experiment, we scale all error parameter simultaneously, with base parameter r_s where $r_{\Delta v} = r_s / 5$ and $r_{\dot{v}} = r_s \cdot 2$. For the reaction time we use t_r = {0.0s, 0.1s, 0.2s, 0.3s, 0.4s, 0.5s}.

Due to reaction time stochasticity as mentioned above, these reaction times are lower bounds. Reaction times vary amongst drivers and can be 0-0.5s larger. Additional to these 36 scenarios, we use 1 'average scenario' with $r_s = 0.05$ and $t_r = 0.25$s for more in-depth comparisons. Note that the deterministic model is given by the scenario with $r_s = t_r = 0$.

4.2 Performance indicators

In the experiment, we aim to evaluate the effect of driver perception through reaction times and perception errors on traffic flow and safety. We consider *capacity* (max flow sustained for five minutes at the most downstream bottleneck)*; time-to-collision* (TTC) *frequency*, *extreme deceleration frequency* (EDF) and *accident frequency rate* (AFR). The AFR indicator is defined as the total amount of kilometers driven by all vehicles in a simulation run before a collision occurs.

For each scenario 3 replications are performed. For the safety indicators, this is sufficient, since each run gives many thousands of inter-vehicle observations. We are aware that for estimating capacity, more runs are needed, which due to technical problems was not possible before publication. Note that 3 random arrival patterns of vehicles and their characteristics will result in the 3 replications, but these arrival patterns are equal for all scenarios for a given replication number. The random numbers used for perception do not affect the arrival pattern. This increases comparability of scenarios.

5 Results

5.1 Traffic flow performance

No anticipation a) | Constant speed b) | Constant acceleration c)

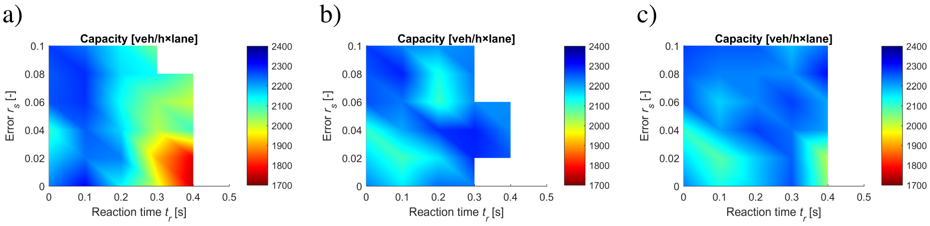

Fig. 5: Traffic flow performance (capacity) for three anticipation scenarios. Note that reaction times are lower bound values; drivers may experience up to 0.5 second additional reaction time

Fig. 5 visualises capacity values for each anticipation strategy. Note, that results are only given for scenarios in which no collisions have taken place between vehicles.

If in all three replications a collision took place, then no result is produced. Otherwise, the figures show the mean value of all available simulation runs. Inclusion of anticipation in driver's behavior can be seen to lead to an improvement in capacity values as well as TTS for increasing reaction times, especially above an initial reaction time value of 0.2 seconds. There doesn't appear to be any substantial difference between the two anticipation strategies. Given the limited number of replications these results must be taken with reservations.

5.2 Traffic safety results

The results of the traffic safety indicators: time-to-collision (TTC) and extreme deceleration frequency (EDF) are given in Fig. 7a-b. The accident frequency rate (AFR) results are given in Fig. 8a-c.

The results of the TTC and EDF are shown as probability distributions normalized to lane-kilometers and hours. Fig. 7a, shows the constant acceleration strategy performs better than the constant speed strategy, and almost as good as the deterministic model. Without anticipation there are more very small TTC values (1 – 2s). However, higher but still critical TTC values occur much less frequent. This is not a direct result from the lack of anticipation. Rather, this is a result from the very different congestion pattern in this scenario as shown in Fig. 6b (only two very dense jam waves emerge in this case). With a reduced number of shockwaves, there are simply less fluctuations in speed. A similar performance for safety is found when the EDF is reviewed in Fig. 7b. The EDF performance for no anticipation strategy performs the worst, as may be expected. Anticipation reduces the frequency of extreme decelerations. Again constant acceleration performs close to the deterministic model, and better than the constant speed strategy.

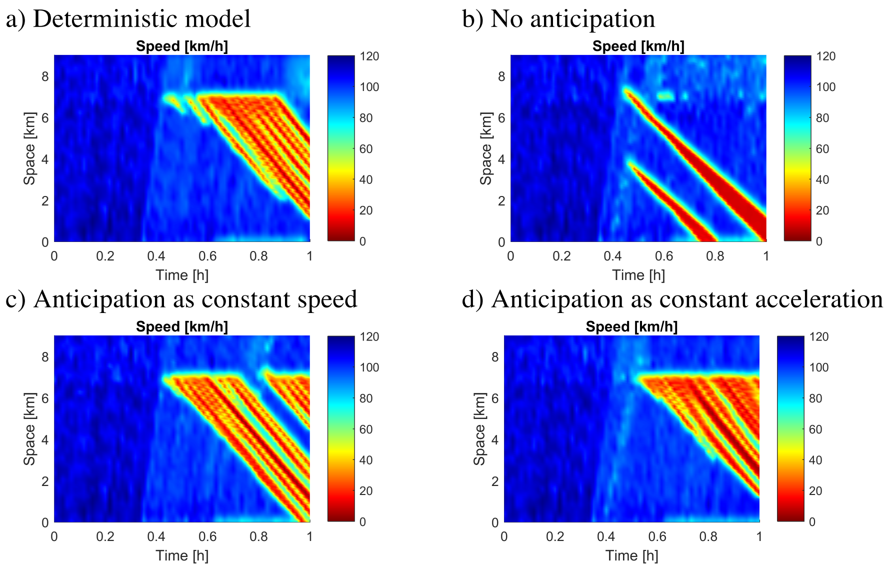

Fig. 6: Congestion patterns for different anticipation scenarios

a) b)

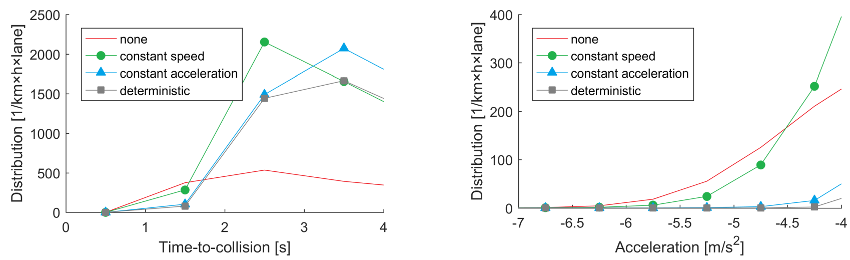

Fig. 7a-b: Traffic safety performance indicators per anticipation strategy; TTC and EDF

The accident frequency rate (AFR) results show an interesting difference between the three anticipation strategies. The accident rate is the highest (i.e. most kms per accidents) for the anticipation scenario with a constant speed, with the anticipation scenario with constant acceleration anticipation also scoring relatively high compared to no anticipation. Reaction time has a clear influence on the accident rates, with higher reaction times leading to a greater chance of accidents. Again, the perception error does not seem to have any significant influence on the probability of an accident.

No anticipation a) Constant speed b) Constant acceleration c)

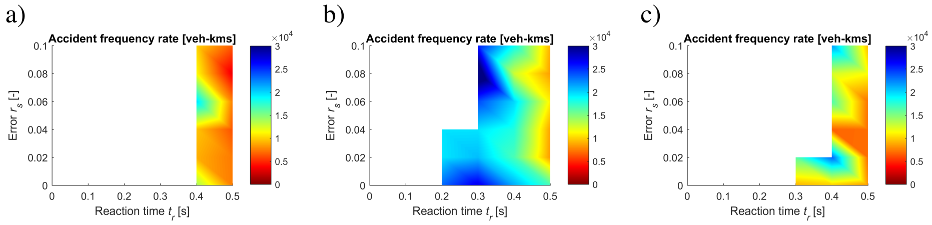

Fig. 8a-c: Accident frequency rates for three anticipation scenarios. Note that reaction times are lower bound values; drivers may experience up to 0.5 second additional reaction time.

6 Discussion and Synthesis

Given our assumptions about collision-free driving (the LMRS model)—the results show that safe driving is fairly robust against the entire range of perception errors we tested under simple anticipation strategies and reaction times of up to a second. Anticipation does *mostly* cancel the effects of reaction time, although more critical situations and more collision occur. The simplest strategy, constant speed anticipation performs slightly worse than constant acceleration anticipation. The latter shows results that are overall close to a deterministic model, that is driving with no perception errors and no reaction time.

There are, however, some limitations. First, we have used a limited number of replications—time restrictions prevented us from doing more. This makes inference on capacity debatable. Second, humans are not good at estimating accelerations. We have incorporated this with a large perception error, but see no significant influence of these large errors. One probable reason for this is that the anticipation time considered in this paper is relatively short—just the time needed to bridge the reaction time delay. When we consider anticipation over longer time periods we hypothesize that more advanced anticipation strategies would pay off.

Future work will focus on testing the same hypotheses in more demanding scenario's in which also scanning frequency (attention span) is taken into consideration.

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